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Date - 01/06/2024

B.Sc. Part II Paper IV

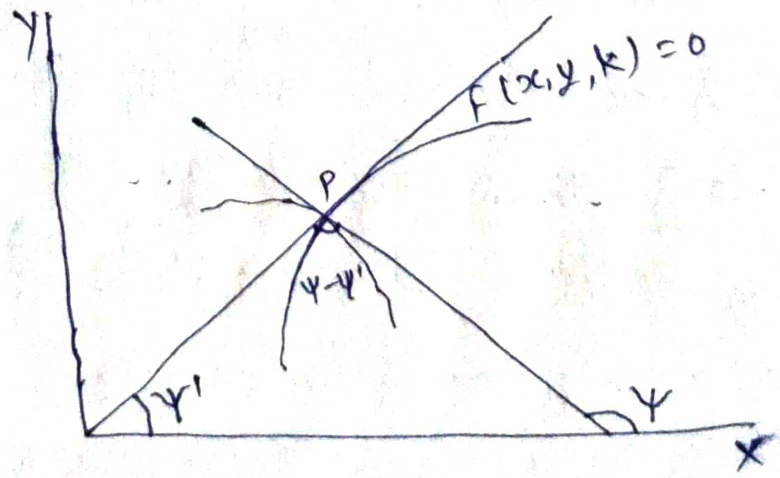
Differential Equation (Orthogonal trajectory)

A curve which cuts every member of a given family of curves according to a given rule is called a tranjectory of the given family.
we shall be concerned only with the case when each trajectory cuts every member of a given family of curves at a constant angle.
If the constant angle is a right angle, then the trajectory is said to be orthogonal.

Rule for finding the orthogonal trajectory to a given family of curves when its equation is given in -

1. Cartesian co-ordinates

To find the orthogonal trajectories of the family of curves $f(x, y, c)$; c being a parameter.



The equation of the curve is given to be

$$f(x, y, c) = 0 \quad \text{----} \quad (1)$$

Differentiating (1) w.r.t. x and eliminating c, between (1) and the derived result, we shall obtain the differential equation of the given

family of curves. The differential equation obtained will evidently involve x, y and $\frac{dy}{dx}$.
Let the differential equation be

$$\phi\left(x, y, \frac{dy}{dx}\right) = 0 \quad \text{------} \quad (2)$$

Let $F(x, y, k) = 0$ be the eqn. of the curve which cuts the given family $f(x, y, c) = 0$ at a constant angle.
We know that the angle between the curves is equal to the angle between the tangents at their point of intersection.
Let P be the point of their intersection.
Let $m\left(= \tan\psi = \frac{dy}{dx}\right)$ and m' denote the slope of the tangents to the curves $f(x, y, c) = 0$ and $F(x, y, K) = 0$ respectively.

Then $\quad \tan\alpha = \frac{m - m'}{1 + mm'}$

If $\alpha = 90^\circ$, i.e. if the trajectories are orthogonal, then $1 + mm' = 0$.

$$\therefore \quad mm' = -1 \Rightarrow m' = -\frac{1}{m} = -\frac{1}{\frac{dy}{dx}} = -\frac{dx}{dy}$$

Thus if $\frac{dy}{dx}$ is the slope of the tangent of the given family of curves, then the slope of the trajectories would be $-\frac{dx}{dy}$.

Hence the differential equation of the family (of curves), ~~the~~ of orthogonal trajectories; from (2), is

$$\phi\left(x, y, -\frac{dx}{dy}\right) = 0.$$